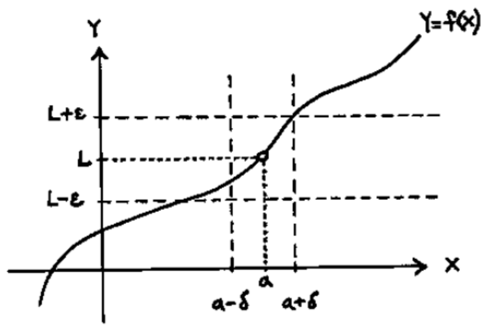
Review: Limit Properties - 10/10/16

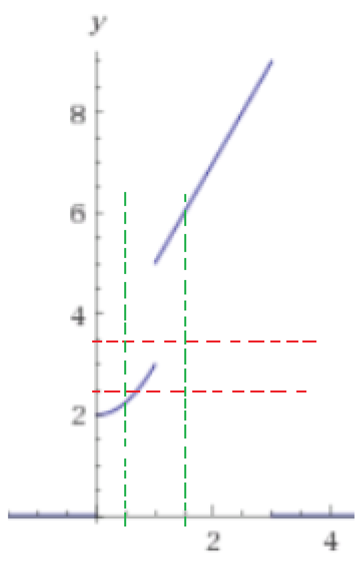
1 Limit Definition

Definition 1.0.1 *We have that* $\lim_{x\to a} f(x) = L$ *if for every* $\varepsilon > 0$*, there exists a* $\delta > 0$ *such that* $|f(x) - L| < \varepsilon$ *whenever* $|x - a| < \delta$*.*

This means that if we make an ε window around our limit L, then we can find a matching δ window around a so that if we pick an x in the δ window, then $f(x)$ will be in the ε window.



Below is a picture showing the problems if the limit doesn't exist.



2 Properties of Limits

Let's look at some properties of limits. Luckily, limits behave as you might hope that they would! For the following, let c be a constant, and let $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist.

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$

4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

5. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

Example 2.0.2 *What is* $\lim_{x\to 2}[3f(x)+g(x)]$*? We can rewrite as* $\lim_{x\to 2} 3f(x) + \lim_{x\to 2} g(x) = 3\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 3(2)+0 = 6$.

3 More Properties

Let n be a positive integer.

6. $\lim_{x\to a}[f(x)]^n = \left(\lim_{x\to a} f(x)\right)^n$

7. $\lim_{x\to a} c = c$

8. $\lim_{x\to a} x = a$

9. $\lim_{x\to a} x^n = a^n$

10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ (If n is even, then we require $a > 0$)

11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ (If n is even, then we require $\lim_{x\to a} f(x) > 0$)

Example 3.0.3 *Find* $\lim_{x\to 4} x^2 + 7\sqrt{x} - 5$. *We start with applying addition and subtraction rules (and the constant multiple one) to get* $\lim_{x\to 4} x^2 + 7\lim_{x\to 4}\sqrt{x} - \lim_{x\to 4} 5$. *Then we can apply rules 7, 9, and 10 to plug in the value for x, getting* $4^2 + 7(\sqrt{4}) - 5 = 16 + 14 - 5 = 25$.

Example 3.0.4 *Find* $\lim_{x\to 3} \sqrt[3]{x^2 - 10}$. *First we use rule 11 to get* $\sqrt[3]{\lim_{x\to 3} x^2 - 10}$, *then the subtraction rule to get* $\sqrt[3]{\lim_{x\to 3} x^2 - \lim_{x\to 3} 10} = \sqrt[3]{9 - 10} = \sqrt[3]{-1} = -1$.

Example 3.0.5 *Find* $\lim_{x\to 3} \sqrt{x^2 - 10}$. *We follow the same process to get* $\sqrt{-1}$. *But that isn't a real number, so the limit does not exist.*

Practice Problems

1. Evaluate $\lim_{x \to 7} \sqrt{x^2 - 4x + 7}$.
2. Evaluate $\lim_{x \to -3} \sqrt[4]{x^3 + 5}$.
3. Evaluate $\lim_{x \to 2} \frac{x^3 - 7x}{x + 4}$.
4. Evaluate $\lim_{x \to 0} \frac{x - 2}{\sqrt{x} + 4}$.
5. Evaluate $\lim_{x \to -1} \sqrt{\frac{x}{x - 4}}$.

Solutions

1. $\sqrt{28}$
2. Limit does not exist.
3. -1
4. $\frac{-1}{2}$
5. $\sqrt{\frac{1}{5}}$